The calculations show a trend whereby the value of H^*/H decreases as Mach numbers increase. Also, the value of H^*/H for base flows with turbulent separating boundary layers are of the order of unity and show a trend similar to, but away from, laminar ones. Results shown in Fig. 2 would therefore suggest that Tanner's assumption that H^*/H being constant, is at best approximate, and his suggested value of $H^*/H \approx 7$ would yield correct values of base pressures only at low supersonic Mach numbers.

If we assume the lipshock to be of significant strength, then flow geometry of the nearwake is necessarily altered. Consequently, an effective shift of the wake shock toward the base results in a decrease of entropy flux due to wake shock in the real flow, as opposed to that in a purely inviscid flow. This forward shift of the wake shock may be perceived as a measure of entropy flux of shear layer and lipshock.

This is still consistent with the Oswatitsch's theorem relating entropy flux to drag. The schlieren photograph of Fig. 3 in Ref. 5 clearly shows a forward translation of wake shock in the real flow. An analogous situation is presented in incident shock/flat plate boundary-layer interactions where the reflected shock wave shifts upstream of the equivalent inviscid position.

It seems plausible, therefore, that there are two mechanisms operating in the forward shift of wake shock in the base flow: (1) the flow geometry is changed by overexpansion of the flow, and (2) the reattaching shear layer-lipshock interaction.

Conclusion

Tanner's theory neglects lipshocks, but he has implicitly accounted for their influence in the ratio H^*/H by equating Eq. (1) with an experimental result. This would explain why Eq. (1) gives a reasonable approximation to $C_{pb}(M_\infty)$. Yet there seems to be no theoretical justification for this. In fact, for turbulent base flow, it seems more appropriate to consider that H^*/H is of the order unity, and available experimental data support this view.

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Source Term Decomposition to Improve Convergence of Swirling Flow Calculations

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Introduction

SWIRLING flows are commonly encountered in aerospace engineering applications. Typical examples are flow in a gas turbine combustor, trailing vortex flow behind an aircraft, etc. The governing differential equations for these flows are the continuity and the Navier–Stokes equation, and in the numerical calculation of such flows, two factors have an important effect on the convergence characteristics of the numerical scheme. The first is the pressure–velocity coupling and the method used to resolve this coupling. The second is the coupling between the momentum equations through their respective source terms. Although the treatment of the pressure–velocity coupling has been systematically studied in the literature,[1–5] methods to resolve efficiently the source term coupling between the momentum equations have not been given the same degree of attention. However, in swirling flows, the source terms, and in particular, the centrifugal term $\rho w^2/r$ in the radial momentum equation become important, and if the source term coupling is not properly addressed, rather poor convergence characteristics of the numerical scheme is noted. The objective of this technical note is to present a source term decomposition technique for the radial momentum equation which greatly enhances the convergence properties of the calculation method and thus reduces the computational time.

Governing Equations

The governing differential equations for steady, laminar, axisymmetric, incompressible flow can be written as

Continuity:

$$\frac{\partial u}{\partial x} + \frac{1}{r}\frac{\partial (rv)}{\partial r} + \frac{1}{r}\frac{\partial w}{\partial \theta} = 0 \tag{1}$$

Axial momentum:

$$\rho \mathbf{u} \cdot \nabla u = -\frac{\partial p}{\partial x} + \mu\left[\frac{\partial^2 u}{\partial x^2} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)\right] \tag{2}$$

Radial momentum:

$$\rho \mathbf{u} \cdot \nabla v = -\frac{\partial p}{\partial r} + \mu\left[\frac{\partial^2 v}{\partial x^2} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v}{\partial r}\right)\right] - \frac{\mu v}{r^2} + \frac{\rho w^2}{r} \tag{3}$$

Tangential momentum:

$$\rho \mathbf{u} \cdot \nabla w = \mu\left[\frac{\partial^2 w}{\partial x^2} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right)\right] - \frac{\mu w}{r^2} - \frac{\rho v w}{r} \tag{4}$$

Received April 20, 1987; revision received Aug. 30, 1987.

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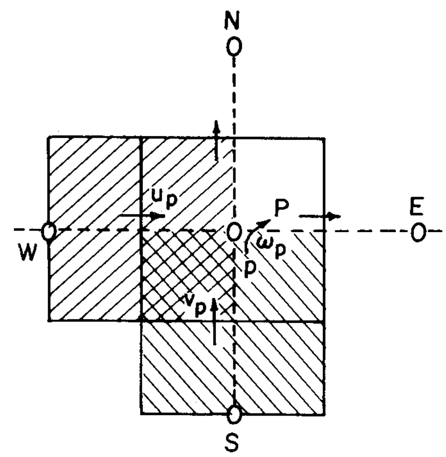

Fig. 1 Schematic of a control volume.

In discretizing the above equations, the control volume-based finite-difference procedure described by Patankar[6] is used. In this procedure, the domain is subdivided into control volumes each associated with a grid point (Fig. 1). The momentum equations are then integrated over each control volume, and by using Green's theorem and profile approximations in each coordinate direction, the axial, the radial, and the tangential momentum equations can each be reduced to the following system of algebraic equations,

$$(a_P - S_P \Delta V_P)\phi_P = a_E\phi_E + a_W\phi_W + a_N\phi_N + a_S\phi_S + S_C\,\Delta V_P \tag{5}$$

or

$$\tilde{a}_P\phi_P = \Sigma a_{nb}\phi_{nb} + S_C\,\Delta V_P \tag{6}$$

where ϕ represents either u, v, or w, and a_P, a_E, a_W, a_N, and a_S are the corresponding convection–diffusion coefficients that depend on the nature of the profile approximation made in each coordinate direction. The terms S_C and S_P arise from a source term linearization as $S = S_C + S_P\phi_P$, and ΔV_P is the volume of the control volume for the grid point P. Equation (5) is rewritten, in a more compact form, as Eq. (6) where the subscript nb refers to the four neighbors of the grid point P (i.e., E, W, N, and S in Fig. 1) and the summation is done over these four neighbors. The system of algebraic equations [Eq. (6)] is solved iteratively. Generally, it is necessary to use an under-relaxation factor λ to obtain converged solutions. At high swirling rates, rather low values of λ's have to be used to prevent the solution from diverging.

The last two terms in Eqs. (3) and (4) are the source terms of the radial and tangential momentum equations, respectively. The most logical, and commonly adopted, decomposition of these source terms at any grid point P (see Fig. 1) are

$$S_C = \frac{\rho_P (w_P^0)^2}{r_P}, \quad S_P = -\frac{\mu_P}{r_P^2} \quad \text{in the } v \text{ equation} \tag{7}$$

$$S_C = -\frac{\rho_P v_P^0 w_P^0}{r_P}, \quad S_P = -\frac{\mu_P}{r_P^2} \quad \text{in the } w \text{ equation} \tag{8}$$

where superscript 0 denotes the current variable values in storage. As noted earlier, the decomposition in Eq. (7) can lead to poor convergence behavior and even divergence at high swirl rates. An improved source term decomposition method is suggested next.

The method proposed in this technical note uses a corrector form of the discretized tangential momentum equation:

$$\tilde{a}_P w_P = \Sigma a_{nb} w_{nb} - \frac{\rho_P w_P v_P}{r_P}\Delta V_P \tag{9}$$

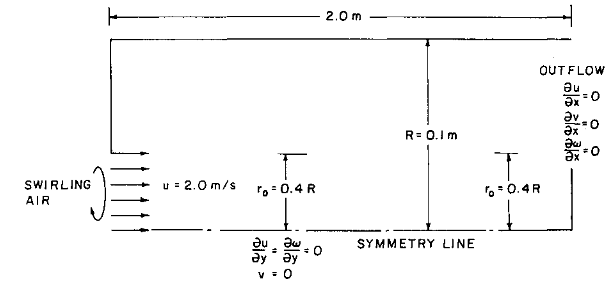

Fig. 2 Isothermal flow in an axisymmetric expansion.

to express the source terms in the radial momentum equation. If, at any iteration, the available solution (in storage) of the implicit system of Eq. (9) is denoted by a superscript 0, then a correction to w can be written as

$$w_P = \frac{\Sigma a_{nb} w_{nb}^0 - \dfrac{\rho_P w_P^0 v_P}{r_P}\Delta V_P}{\tilde{a}_P} \tag{10}$$

If Eq. (10) is substituted for w_P in the discretized source term of the radial momentum equation, the right-hand side of the equation below [Eq. (11)] results. This expression has been written in the desired linearized form of $S_C + S_P v_P$, i.e.,

$$S_C + S_P v_P = \left[\frac{\rho_P}{\tilde{a}_P^2 r_P}\left\{\left(\Sigma a_{nb} w_{nb}^0\right)^2 + \left(\frac{\rho_P w_P^0 v_P^0}{r_P}\Delta V_P\right)^2\right\}\right] - \left[\frac{2\rho_P}{\tilde{a}_P^2 r_P}\left\{\left(\Sigma a_{nb} w_{nb}^0\right)\cdot \rho_P w_P^0\,\Delta V_P\right\} + \frac{\mu_P}{r_P^2}\right] v_P \tag{11}$$

Thus, the first square-bracketed term in Eq. (11) is taken to be the S_C term, while the second square-bracketed term with the negative sign is the S_P term of the radial momentum equation.

It should also be noted, that in the calculation procedure described in Ref. 6, the location of the axial and radial velocities are staggered, as shown in Fig. 1. This is done in order to avoid checkerboard velocity and pressure fields. Therefore, the storage location of the tangential and radial velocities are different, and to evaluate the tangential velocities on the right-hand side of Eq. (11), it is necessary to interpolate. In obtaining the results shown in this technical note, it is found advantageous to stagger the location of w in the radial direction, so that w and v are stored at the same location. With this practice, interpolation is not necessary in evaluating the source terms, and this approach is therefore used in the present work.

Results and Discussions

The performance of the source term decomposition in Eq. (11) is examined by comparing the results with those obtained by using the conventional source term decomposition in Eq. (7). For this purpose, the test problem of isothermal flow in an axisymmetric sudden expansion (Fig. 2) is chosen. Slug flow is assumed at the inlet with a uniform swirl velocity characterized by a swirl number S defined as

$$S = \left[\int_0^{r_0} \rho u w r^2\,\mathrm{d}r \Big/ \int_0^{r_0} \rho u^2 r R\,\mathrm{d}r\right]_{\text{inlet}} \tag{12}$$

where r_0 and R are the radii of the inlet opening and of the axisymmetric chamber, respectively. Results are obtained for a low ($S = 0.5$) and a high ($S = 2.0$) value of the swirl number.

Calculations are made with a 10×30 grid on an IBM 3084 machine. Iterations are continued until the following residual convergence criterion is satisfied

$$\left[\|r_u\|^2 + \|r_v\|^2 + \|r_w\|^2\right]^{\frac{1}{2}} \leq 10^{-5} \tag{13}$$

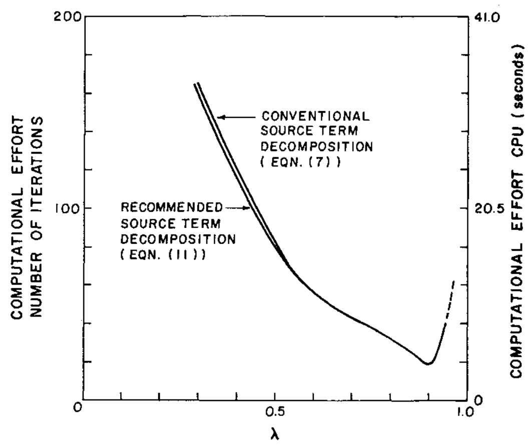

Fig. 3 Comparison of computational effort for swirl number $S = 0.5$.

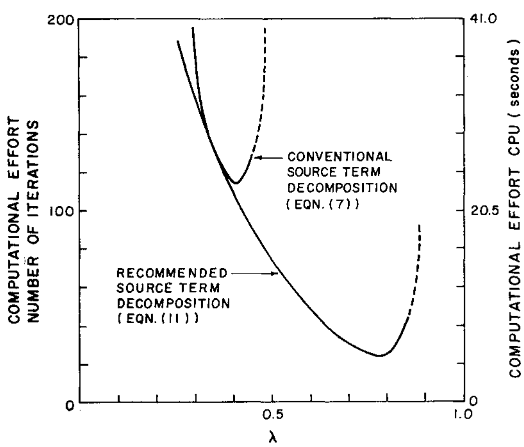

Fig. 4 Comparison of computational effort for swirl number $S = 2.0$.

where $\|r_u\|$ is the Euclidian norm of the residual for u, and it is defined as

$$\|r_u\| = \left[\sum_{C\cdot V}\left(\sum_{nb} a_{nb}u_{nb} + S_C\,\Delta V_P - \tilde{a}_P u_P\right)^2\right]^{\frac{1}{2}} \tag{14}$$

The outer summation in the preceding equation is performed over all the control volumes in the domain.

Results are presented in Fig. 3 for $S = 0.5$, and in Fig. 4 for $S = 2.0$. The computational effort expressed as the number of iterations (shown on the vertical left axis), and the CPU time in seconds (shown on the vertical right axis) are plotted against the under-relaxation factor λ.

At low swirl rates ($S = 0.5$), both the conventional source term decomposition [Eq. (7)] and the source term decomposition recommended in this technical note [Eq. (11)] exhibit similar behavior (Fig. 3). However, at the higher swirl rate ($S = 2.0$), the advantages of the recommended source term decomposition [Eq. (11)] are clearly evident. A five-fold decrease in computational effort is obtained with the source term decomposition proposed in Eq. (11). More importantly, the solution algorithm exhibits stable convergent behavior over a wide range of under-relaxation factors (for λ nearly up to 0.8).

Based on the aforementioned comparisons, it is recommended that in swirling flows the source term decomposition in Eq. (11) should be used. A considerable decrease in the computational effort will be obtained when applied.

Concluding Remarks

A source term decomposition method is proposed, and it is shown to be computationally advantageous in highly swirling flows. The proposed method uses a corrector form of the tangential momentum equation to decompose the source terms of the radial momentum equation.

Acknowledgments

This work was supported by DOW Chemical Company, Plaquemine, LA. Their support is gratefully acknowledged.

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Passive Venting System for Modifying Cavity Flowfields at Supersonic Speeds

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Nomenclature

C_D = drag coefficient, based on cavity rear face area, 0.848 in.^{2} (5.47 cm^2)
C_P = pressure coefficient
d = chamber height
h = cavity height, 0.40 in. (1.02 cm)
l = cavity length
M = Mach number

Introduction

EXISTING data available in the literature[1-4] show there are two fundamentally different types of cavity flowfields at supersonic speeds depending primarily on the cavity length-to-height (l/h) ratio. For $l/h \gtrsim 13$, the flowfield expands over the cavity leading edge, attaches to the cavity floor, and exits ahead of the rear face (see Fig. 1). For $l/h \lesssim 11$, the flow passes over the cavity without appreciable deflection. These two flow patterns are generally referred to as closed and open cavity flow, respectively.

A recent experimental investigation[5] has shown that the drag of a cavity with closed flow was substantially higher than that of a cavity with open flow. The higher drag for the closed flow case is due to the large pressure difference between the forward and aft sections of the cavity. In addition, stores

Received Aug. 31, 1987; revision received Oct. 13, 1987.

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